Simulation of Human Migration Based on Swarm Theory

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***Abstract*—The analysis documented in this paper was completed as part of a larger multi-component simulation effort to evaluate the efficiency of sensor networks in detecting and identifying human subjects remotely. Such an assessment necessitates the accurate simulation of movement of people along roads, known paths or over a more open terrain. The individuals are assumed to be traveling in groups of varying size and usually with a designated leader. The mass motion toward a "goal" or away from an "obstacle" is modeled using swarm theory, which successfully applies basic physical principles to the simulation of emergent collective behavior of biological organisms. Individuals are modeled as particles whose velocities are updated using either dynamic or kinematic schemes. In the present analysis, swarming techniques were found to reproduce quite well the group movement and behavior of individuals over predefined surface conditions, as determined by driving forces, social interactions and environmental conditions.**[1]

I. Introduction

There are many areas in which the detection and identification of humans at a distance becomes necessary, such as military operations, law enforcement or search and rescue efforts. In order to evaluate the performance of the sensor systems and the accompanying algorithms designed to accomplish this, it is important to model the movement of people as accurately as possible. The focus of this work is to model the collective behavior of self-propelled individuals moving in a coordinated manner. It is significant because it provides us with a mechanism to quickly and easily recreate a limited, but desired, set of movement patterns that could be used to stimulate a simulated network of sensors.

Global organization and pattern formation are observed everywhere in nature, e.g., swarming, flocking, herding, among insects, fish, mammals and other organisms. However, swarming concepts have wide applicability not only in the field of biology, but in numerous areas as diverse as computer science, engineering, transport phenomena such as traffic patterns, animation and of course physics from which the theoretical underpinnings arise, [1]. Studies of cooperative behavior have been conducted in a variety of mathematical frameworks, [2], [3], [4]. As in the present work, one approach is to treat individuals as discrete particles obeying simple rules of motion, driven by self-propulsion, frictional forces, interactions with other particles in the group and random causes. In addition, the directional drift may be determined by attraction toward a goal or leader and/or repulsion away from obstacles or hostile agents. The particle positions and velocities are found through the solution of a set of nonlinear equations of motion. For numerical computation, the equations are discretized and updated at each predetermined time step. Environmental influences may be added through terms proportional to the gradient of the terrain at the location of each particle. The gradients in turn may be obtained from elevation maps of the area of interest. Steep uphill or downhill slopes, such as hills and canyons, are programmed to impede the particle motion. Other studies have been based on continuum settings, [4], with physical quantities being modeled as scalar or vector fields. One widely used way to construct a continuum model is by coarse-graining a discrete particle model, [5].

Numerical simulations clearly reveal collective behavior and global coordination, even if direct human contact through speech is not included. However, because of the nonlinear nature of the equations of motion, slightly differing initial conditions may lead to widely varying evolutions of the swarm structure across the terrain, unless tight boundary conditions are imposed. In this paper, we focus on two desired movement patterns. The first addresses a collective movement pattern for a crowd ($\geq$ 100 individuals). The second pertains to the movement of a small group ($\leq \sim 20$) of individuals following a decision maker along a well defined route.

Section II of this paper contains a description of the theory underlying our swarming method. Section III provides examples for various patterns of motion across the terrain of interest, and Section IV summarizes our conclusions.

II. Swarming Theory

Swarming models may be classified as dynamic or kinematic, depending on how the particle velocities are determined. We have used both approaches, as described below.

A. Dynamic Model

In dynamic models, velocities are updated at each time step using Newton's second law of motion. The discrete model in this work is two-dimensional and consists of self-propelled interacting particles. Assume that a group consists of N particles representing an equal number of agents with mass m_i, position $\overrightarrow{x_i}$ (x and y components) and velocities $\overrightarrow{v_i}$.

[1]Approved for Public Release: 10-0753. Distribution Unlimited.

The particles move forward through a self-propelling force $\overrightarrow{f_i}$ with fixed strength α, while a frictional force with coefficient β may be added to prevent the particles from attaining large speeds. Cohesion among group members is ensured through attractive two-body forces, while shorter-range repulsive two-body forces prevent a collapse of the swarm. The vector equations of motion may then be written as:

$$\begin{aligned} m_i \partial_t \overrightarrow{v_i} &= \alpha \hat{f}_i - \beta \overrightarrow{v_i} - \nabla U + \gamma \hat{f}_g + \delta \hat{f}_r, \\ \partial_t \overrightarrow{x_i} &= \overrightarrow{v_i}, \\ U &= \sum_{j \neq i} C_a \exp\left(\frac{-|\overrightarrow{x_i} - \overrightarrow{x_j}|}{\ell_a} \right) \\ &- \sum_{j \neq i} C_r \exp\left(\frac{-|\overrightarrow{x_i} - \overrightarrow{x_j}|}{\ell_r} \right), \end{aligned} \tag{1}$$

where ∂_t denotes a partial derivative with respect to time and a caret over a symbol signifies a unit vector. It has been verified that the qualitative value of the simulation results does not depend on the exact mathematical form of the potential, [5]. Here, we have chosen as a physically reasonable potential, the quantity U consisting of exponentially decaying attractive and repulsive terms of strength C_a, C_r and interaction range ℓ_a, ℓ_r, respectively. The symbols γ, δ represent magnitudes or scale factors for the additional forces. The direction of the self-propelling force may be chosen along the instantaneous velocity vector or along the average velocity direction of neighboring particles:

$$\begin{aligned} \hat{f}_i &= \hat{v}_i, \\ \text{or} \qquad \hat{f}_i &= \sum_{j \neq i} \hat{v}_j \exp\left(\frac{-|\overrightarrow{x_i} - \overrightarrow{x_j}|}{\ell_c} \right), \end{aligned} \tag{2}$$

where ℓ_c is a correlation length. The unit force $\hat{f}_g$ is an estimate of the gradients in the x and y directions computed from an elevation map of the area of interest. The map file consists of a large grid from which a smaller sector may be chosen to reduce computational costs and for easier visualization of scenarios on scales compatible with the sizes of the swarms. In our simulation, each gridpoint represents a 40 m by 40 m area. Aside from the issue of computational intensity, the map file may be, in general, constructed from a much larger number of gridpoints. The grid density can also reflect environmental effects, in addition to detail of motion. The gridpoint separation may be thought of as correlating roughly to line-of-sight visibility, thus enabling an exploration of micro versus macro terrain effects. For instance, a foggy day might equate to closer gridpoint separation. Therefore, particles move toward a goal but only react to terrain variations that are immediately upon them, with no longer term capability. Conversely, a clear day might equate to greater gridpoint separation. Particles can "see" terrain variations from further away as they move toward a goal, and can adjust their path for greater efficiency.

The gradient at each gridpoint is computed by taking the differences in elevation H, dH/dx dH/dy, in the x or West-East and y or South-North directions. The gradients at the individual particle locations are then found through two-dimensional interpolation. Although the elevation magnitudes are given at gridpoints in terms of the latitude and longitude, these may easily be converted to linear distance scales by computing distances between points on the WGS-84 ellipsoidal Earth to within a few millimeters of accuracy using Vincenty's algorithm, [6]. The force $\hat{f}_g$ then serves to realistically represent the retarding or quickening effects of the terrain on an individual's movement. A simple example is discussed below. The term $\hat{f}_r$ represents randomness in a particle's motion and varies from individual to individual, as well as from time step to time step.

In addition to the above forces, motion toward a destination point (camp, town, etc.) may be modeled by adding an attractive force of the same form as the interparticle attractive forces of Eq. (1), while motion away from obstacles may be added on as a repulsive force arising from the following potentials:

$$\begin{aligned} Ug_i &= C_g \exp\left(\frac{-|\overrightarrow{x_i} - \overrightarrow{x_g}|}{\ell_g} \right), \\ Uo_i &= - \sum_j C_{o_j} \exp\left(\frac{-|\overrightarrow{x_i} - \overrightarrow{x_{o_j}}|}{\ell_{o_j}} \right), \end{aligned} \tag{3}$$

where C_g is the attractive strength, $\overrightarrow{x_g}$ is the position of the goal, ℓ_g is the attractive interaction range, C_{o_j} is the repulsive strength of the jth obstacle, e.g., hill, $\overrightarrow{x_{o_j}}$ is the position of the jth obstacle and ℓ_{o_j} is the corresponding interaction range. In order to implement Equation (1) numerically, it has been discretized in the simulation algorithm.

Examples of the effect of the ground slope on swarm motion are presented in Figures 1 and 2. In the top plot of Figure 1, we see a group of 100 particles positioned between -100 m and 100 m starting to move across a nonlevel terrain from left to right. In Figs. 1(b)-(e), which correspond to successive steps of the motion, it can be seen that the particles are drawn to the negative "potential well" in blue. In fact, depending on the magnitude of the slope, the particles will face difficulty getting up the other side of the well. In Figures 2(a)-(e), instead of a depression in the ground, there is an elevation and the particles are seen to concentrate in the lower part of the graph in trying to avoid it. It will be noticed that the particles progressively disappear from view off the right edge of the plots as they move from left to right. This type of scenario might arise in situations of crowd movement, where the exact details of the motion are not important, as long as the individuals are seen to move toward an attractor or under the guidance of a leader. Of interest is the extent to which obstacles, attractors or any distractions will influence individuals to veer off from the group movement. A more dispersed sensor system would be implied here.

The initial positions of the swarm members may be defined by specifying mean x and y coordinates and a random distribution parameter around the mean, such as a standard deviation. Alternatively, the particles may be distributed in some random or deterministic fashion behind a leader whose starting point is given. The particles may also be oriented toward a specific destination point by fixing their mean velocity direction along a line joining the initial mean group position with that point. This proves helpful when the particles need to be confined to a well-defined roadway. The shoulders of the road may be drawn by specifying a “margin” around the mean road direction. If one wishes to completely confine the group inside the road borders, one may set up “potential barriers” by essentially defining the borders as obstacle points and then making the repulsive strength large. Both techniques have been used in this work.

Table 1 in the Appendix provides a brief outline of the general algorithm steps.

B. Kinematic Model

In a kinematic model, inertial effects do not play a role, since the particle motion is not determined by the forces acting on them, e.g., [7]. For continuum models, the velocity is simply a functional of the population density: $\vec{v} = \vec{V}(\rho)$. The functional may encompass effects such as self-propulsion, social interactions and influence of the surroundings. In this work, which is based on a discrete formulation, we have adhered to a kinematic model to approximate the motion of a company of travelers along an arbitrarily shaped road. The road coordinates are contained in a data file that is loaded into the program. We have designed a simple algorithm, where the particles follow one another such that their positions at each iteration are randomly distributed about successive points along the road with a small standard deviation. The leader is always at the head of the procession, which may be slowed down by increasing the time interval for successive steps or by interpolating multiple times between road point coordinates, so as to obtain a greater point density.

III. Example Results

A. Dynamic Model

In the first example result, a group of five particles, each of mass equal to one unit, starts out at a mean position of x = 60 m, y = -150 m with a velocity of magnitude equal to one unit and a direction along the road linking the mean position with the position of the goal at x = 150 m, y = -80 m. The borders of the road are depicted in white in Figure 3, where the road “margin” has been set equal to 1.5 m. The sampling time is .01 sec, the interparticle attractive strength C_a = 300 units, the interparticle attraction range ℓ_a = 2.6 m, the interparticle repulsive strength C_r = 150 units, the interparticle repulsion range ℓ_r = 2.0 m, the goal attractive strength C_g = 100 units, and the goal attraction range ℓ_g = 300 m. In addition, there are three “obstacles” that the group must avoid at x_j = 125 m, 80 m and 90 m, y_j = -100 m, -140 m, -110 m, respectively. The repulsion strengths are C_{o_j} = 40, 40, and 20 units, while the interaction ranges are ℓ_{o_j} = 300 m, 300 m and 100 m, respectively. Figure 3(a) shows the group starting out far from the obstacles which are depicted as yellow circles. We note that all five members are essentially confined to the road. Figure 3(b) shows the group as it begins avoiding the first obstacle, while 3(c) shows them about halfway to the goal after executing a wide maneuver to avoid all the yellow pitfalls. The fourth plot, Figure 3(d) shows the group curving around toward the location of the goal, shown as a red square. In this case, the group eventually reaches the goal, since the attraction strength is greater than the repulsion experienced by the presence of the three obstacles. The leader is shown as a black circle, while the followers are colored magenta. This type of motion, as compared to that in Figures 1 and 2, is more restricted, as the individuals adhere more closely to the leader’s chosen path. Such a scenario might arise when a small group is expected to follow established roads or rambling footpaths, with the sensors placed at positions closer to the route.

In the second example, Figure 4, two separate groups are shown advancing across part of the terrain at two different times with and without obstacles. Group 1 consists of five people and is following a leader along a well defined path steadily toward a goal at x = 150 m, y = -40 m with no obstacles along the way. The goal attraction strength is 200 units and the attraction range is 300 m. Group 2 has ten individuals not following a specific path and therefore more spread out than Group 1. Their goal at x = 0 m, y = 150 m exerts an attractive force of 200 units with a range of 300 m. Both goal points are depicted as red squares. Figure 4(a) shows the configurations of the groups after 600 iterations through the equations of motion with no obstacles. We see that Group 1 remains confined to the road, which here is ensured by setting the direction of the velocity vector strictly along the road with small variations, and by setting the initial mean position at x = 60 m, y = -150 m. The deviation about the mean position was 1 m, while the road was roughly 10 m wide. Group 2 started out at x = -60 m, y = -150 m and is headed in the direction of its goal at this point in time. If, however, two obstacles are interposed (yellow circles in Figures 4(b) and 4(c)), Group 2 is repelled further to the right and away from the goal at 600 iterations (Figure 4(b)). The repulsion strengths have been set high at 400 units for the obstacle at x = -125 m, y = -125 m, and 800 units for the obstacle at x =-40 m, y = 0 m, to emphasize the effect on the group’s motion. In Figure 4(c), when at 1200 iterations the group is sufficiently far away from the repellers, it starts winding its way around to the goal. The particles of Group 1 have already reached their destination at this point in time. The color bars next to Figures 1, 2 and 4 are elevation scales in meters.

We have seen in the above examples that it is straightforward to ensure that a group of particles follows a straight road by setting a constant velocity along the direction of the road and placing a goal at the end of it. For an arbitrarily shaped road or dirt path, there are several possible ways to guide the relevant group of particles along its course. One is through the use of strategically placed obstacles or walls, as is done in robotics. Figure 5 shows a group of ten individuals confined to a narrow strip of land by two “potential barriers” (lines in black). They are moving north without the presence of an attractive goal. Plot 5(a) shows the particles advancing in single file, while in plot 5(b) they are advancing in double file. This ordering is accomplished by increasing or decreasing the repulsion range of the “walls”, respectively. Another way to ensure motion along a given curve is by using the kinematic method which was discussed in Section IIB above.

Figure 6 depicts a group of forty individuals moving across open terrain with a northerly speed and no obstacles except the elevation slope. In plot (a) the group appears cohesive and moving upwards in unison. In plot (b), the group disperses as it approaches the steep rise at the top of the image. Rather than continue moving upward, the members now disperse as they avoid climbing the hill by spreading their motion around its base.

B. Kinematic Model

An example scenario in which the kinematic model was used is presented in Figure 7. The road coordinates were read in from a separate data base in terms of longitude and latitude. In the case of this example, these were converted to linear x and y distances relative to the geodesic coordinates of the first road point. The five group members are captured at a certain point in time as they advance toward their goal following the leader. We note that their positions lie strictly along the curve of the road. Figure 7(a) is a higher level view, while Figure 7(b) is a close-up around the vicinity of the group. The positions may be made to stray from a thin line by defining road borders parallel to the main curve and allowing the particles to move within the finite width of the road. Alternatively, the particles may be made to diverge from the exact road points by including a random deviation in their successive positions. This will allow for a more realistic representation of individuals wandering slightly off the road boundaries in an independent fashion. Figure 7(c) contains an example of the overall view of a slightly more random motion, while Figure 7(d) contains an enlargement around the group members.

IV. Conclusions

In order to enable accurate performance evaluation of sensor networks in detecting and identifying human subjects, it becomes necessary to realistically simulate the movement of people along roads or across open areas. In this work, the collective orientation and behavior of groups of particles has been modeled using swarm theory. Driven by individual self-propulsion, interactions between swarm members, environmental and random forces, collective behavior emerges which depends critically on the initial conditions. Based on dynamic or kinematic principles, the two-dimensional nonlinear equations of motion are solved by discretization and iteration over a number of time steps. The effect of the topography is included via a slope-dependent force, found by calculating the gradient at each particle location from a given elevation map of the area of interest.

Movement toward a specific location is modeled by defining a goal point which exerts an attractive force on each particle or on a leader and then making the leader the goal of the remaining particles. Movement away from undesirable areas is accomplished by defining obstacle points which exert a repulsive force on the group members. Interparticle, as well as goal-particle and obstacle-particle interactions are modeled by exponentially decaying potentials with interaction strength and range as parameters. Motion on straight roads is most easily reproduced by defining the velocity of the group along the direction between the mean initial position and the position of a goal. Alternatively, “potential barriers” may be set up to restrict group motion within the road boundaries. For arbitrarily shaped road curves, a kinematic approach is used in which the particles follow a leader such that their mean position at each point in time coincides with a road point. The motion may be slowed down by increasing the number of time steps between road points or by increasing the point density through interpolation between the given road point coordinates.

From this initial study, swarm theory appears to reliably replicate a limited but desirable set of movement patterns. Furthermore, selectable attributes (e.g., position and strength of attractors/repellers, initial conditions, etc.) offer a promising capability to recreate an even wider set of movement scenarios. This analysis successfully supported the larger simulation task, that included issues related to movement of people in a sensor field. The next step in this research project will be the simulation of more complicated and larger scale migrations of people in a variety of environments.

Appendix

Load area elevation map; compute terrain gradients (x and y directions); set axis scales;
Input swarm model parameters: Number of : particles, iterations, goals, obstacles; Sampling time; Interparticle attraction and repulsion ranges and magnitudes; Goal and obstacle point positions, potential ranges and magnitudes;
Generate initial swarm position distribution: μ_x, μ_y, σ_x, σ_y, additional randomness;
Generate initial particle velocities: directions, speeds and random deviations; Define leader; Introduce forces between leader and followers;
Define geometry of roads: direction, shape, borders, extent; or read in arbitrary path points from file;
Remove self interactions;
Loop over iterations in time: Compute interparticle distances; Loop over particles; Compute terrain slopes at particle position; Compute interparticle attractive and repulsive gradient components for Gaussian kernels; Compute attractive and repulsive gradients to goals and obstacles; Add forces; Update particle position and velocity; Plot snapshot of scene; End loop; End loop;
Make movie;

Table I
Swarm Simulation Outline.

References

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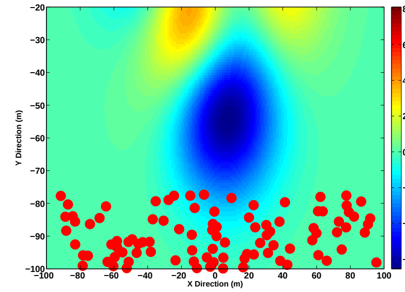

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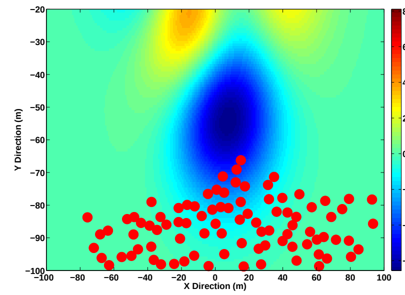

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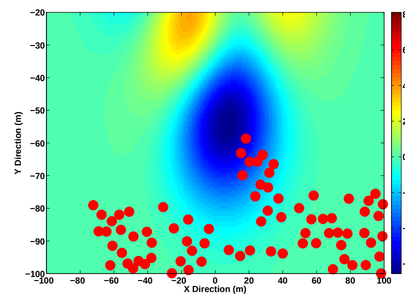

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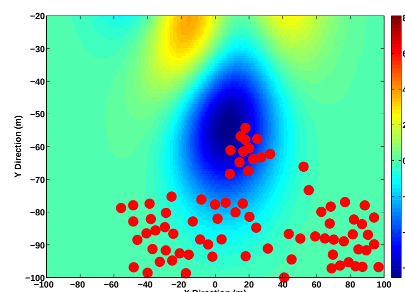

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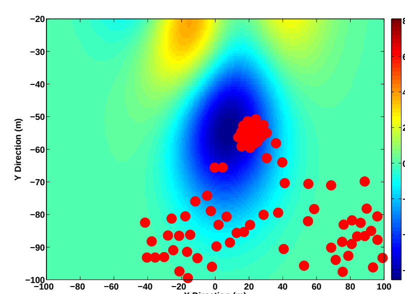
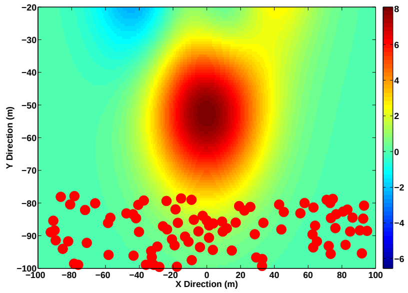


(e)

Figure 1. Effect of Terrain Gradient on Motion with Particles Traversing Area of Negative Slope: (a) Time Step = 10, (b) Time Step = 70 , (c) Time Step = 140, (d) Time Step = 210, (e) Time Step = 300.



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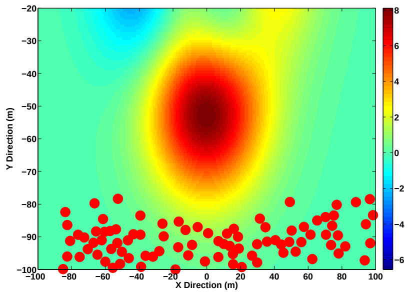

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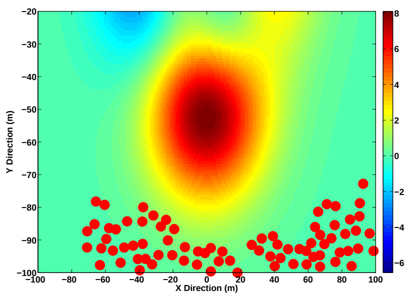

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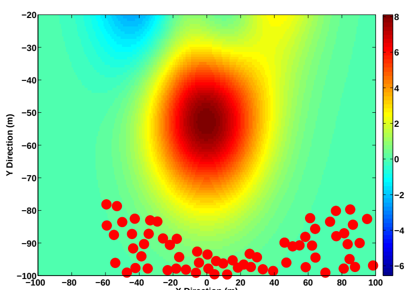

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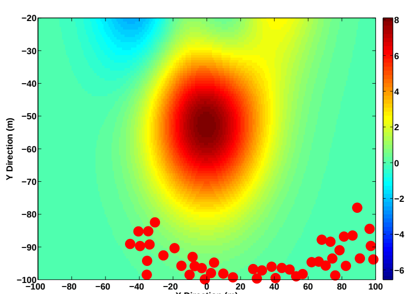
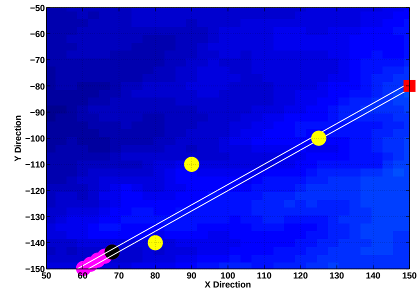


(e)

Figure 2. Effect of Terrain Gradient on Motion with Particles Traversing Area of Positive Slope: (a) Time Step = 10, (b) Time Step = 70 , (c) Time Step = 140, (d) Time Step = 210, (e) Time Step = 300.



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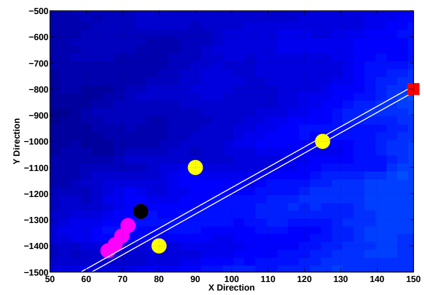

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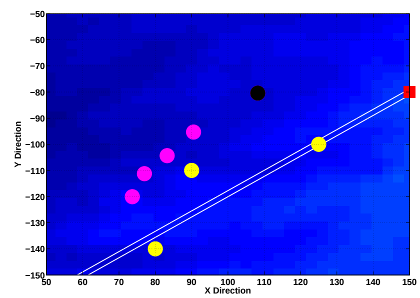

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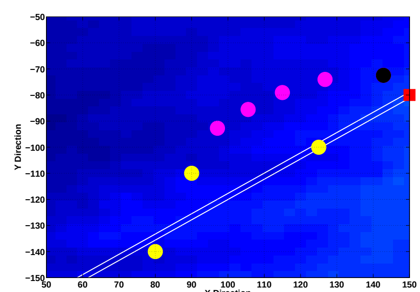
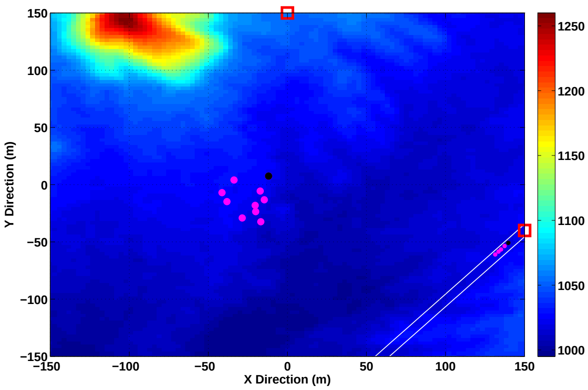


(d)

Figure 3. Motion of Particles Toward a Goal with Intervening Obstacles: (a) Onset of Motion, (b) Initial Sensing of Obstacle, (c) Steering Clear of All Obstacles on Way to Goal, (d) Nearing Goal.



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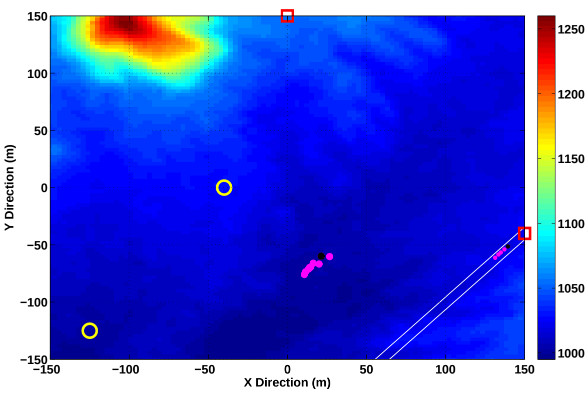

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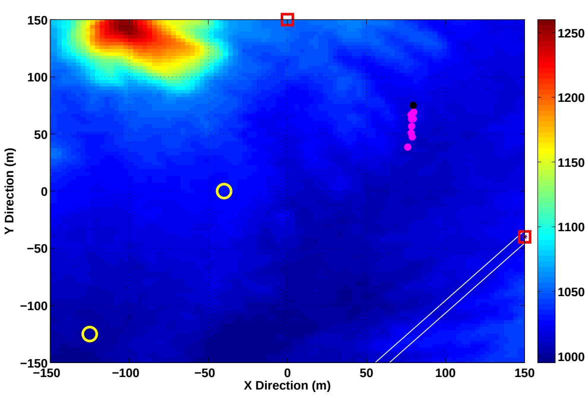

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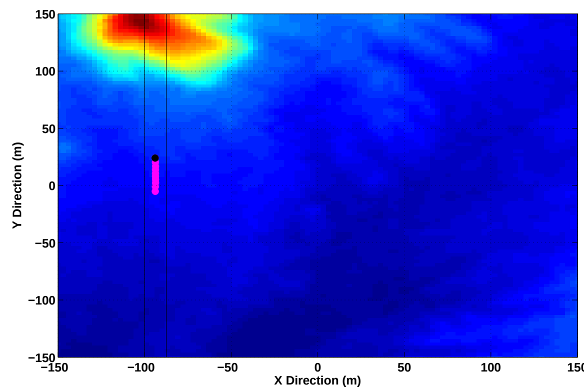
Figure 4. Motion of Two Groups Toward Separate Goals: (a) No Obstacle, 600 Iterations, (b) With Obstacle, 600 Iterations, (c) With Obstacle, 1200 Iterations.



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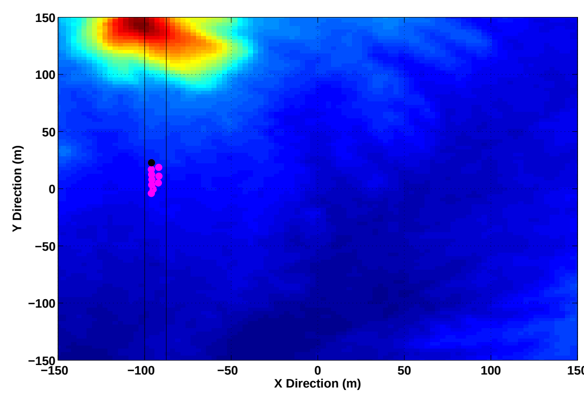
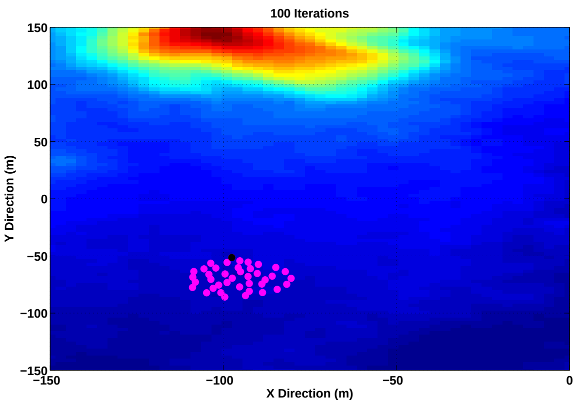


(b)

Figure 5. Group Confined to Narrow Strip of Land: (a) Single File, (b) Double File.



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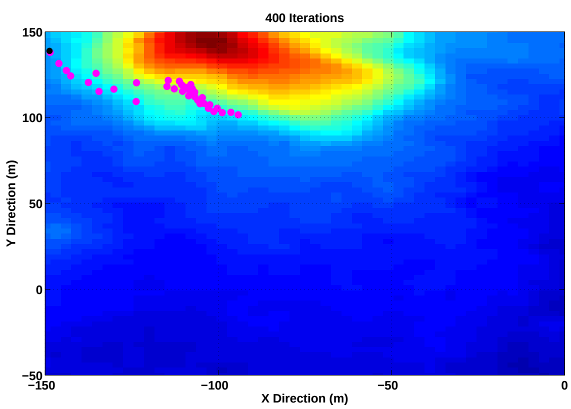
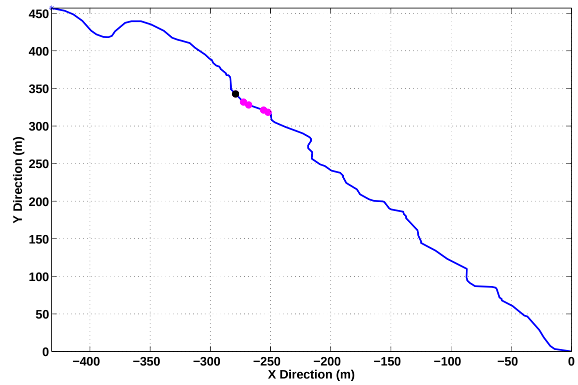
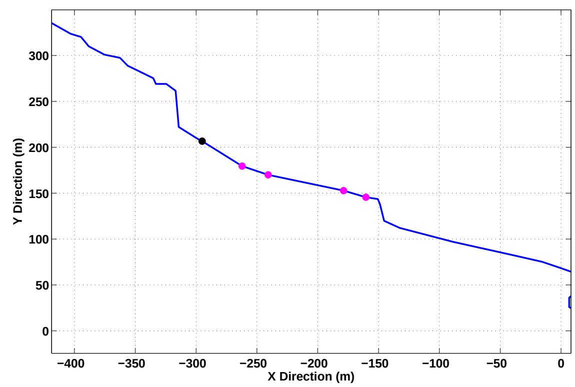


(b)

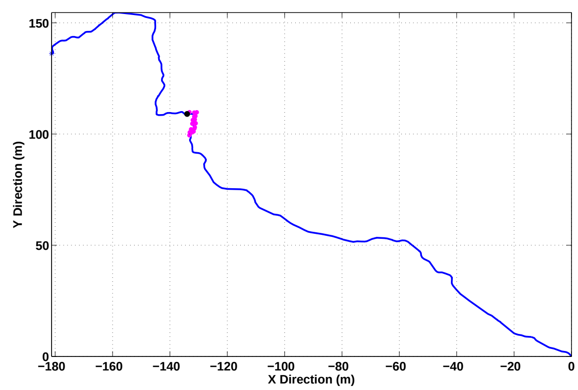
Figure 6. Group Moving Along Open Terrain: (a) Far from Hill, (b) Near Hill.



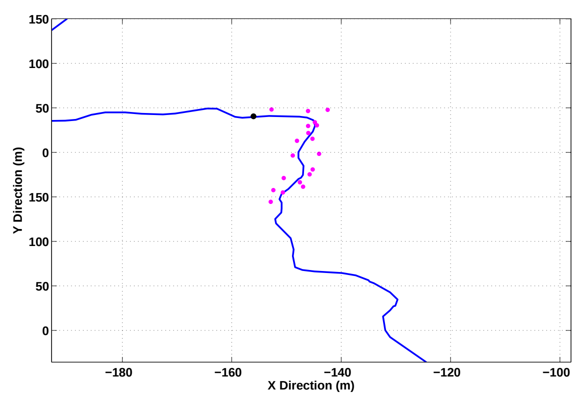
(a)



(b)



(c)



(d)

Figure 7. Particles Advancing Along an Arbitrary Road where Black Circle is the Leader and Magenta Circles are the Followers, (a) No Deviations, Overall View, (b) Closeup, (c) Small Deviations, Overall View, (d) Closeup.